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The Cauchy Singular Integral on Non-Smooth Curve

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Abstract—We consider the Cauchy singular integral with Hölder's density on non-smooth curves: various versions of its definition, conditions for its existence, boundedness and so on.

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Let Γ be a simple Jordan curve on the complex plane $\mathbb{C}$. The original definition of the Cauchy singular integral over this curve is

$$\mathcal{S}_\Gamma f(t) := \lim_{\epsilon\to 0}\frac{1}{\pi i}\int\limits_{\Gamma\setminus\{|\tau-t|\le\epsilon\}}\frac{f(\tau)d\tau}{\tau-t},\quad t\in\Gamma. \tag{1}$$

Of course, the existence of the limit needs the proof.

This integral operator has a lot of applications. In particular, it is of importance for theory of boundary value problems for analytic functions, for aero and hydrodynamics, and for theory of elasticity, see [1–3]. There exists a great body of publications on this subject. Here we restrict our references by the classical monograph [4] and recent survey [5].

It is well known [1–3], that $\mathcal{S}_\Gamma f$ exists if the curve Γ is smooth or piecewise-smooth, and the density f satisfies the Hölder condition

$$\sup\left\{\frac{|f(t_1)-f(t_2)|}{|t_1-t_2|^\nu} : t_{1,2}\in\Gamma, t_1\ne t_2\right\} := h_\nu(f;\Gamma)<\infty;$$

with any exponent $\nu\in(0;1]$. We denote the class of all that functions $H_\nu(\Gamma)$. The norm $\|f||_\nu = \sup\{|f(t)| : t\in\Gamma\} + h_\nu(f;\Gamma)$ turns this class into Banach space.

If $f\in H_\nu(\Gamma)$ and Γ is smooth, then the function $\mathcal{S}_\Gamma f(t)$ is continuous and satisfies the Hölder condition with the same exponent ν for $\nu<1$, and with arbitrarily close to unit exponent for $\nu=1$. But this result is not valid at the corners of a piecewise-smooth curve. For example, if Γ is boundary of a square with counterclockwise circuit and $f\equiv 1$, then function $\mathcal{S}_\Gamma f$ equals 1 at all points of Γ excluding the vertices, where it is equal to $3/2$.

In this connection there arises another definition of the Cauchy singular integral:

$$\mathcal{S}^*_\Gamma f(t) := f(t) + \frac{1}{\pi i}\int\limits_\Gamma\frac{f(\tau)-f(t)}{\tau-t}d\tau,\quad t\in\Gamma, \tag{2}$$

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